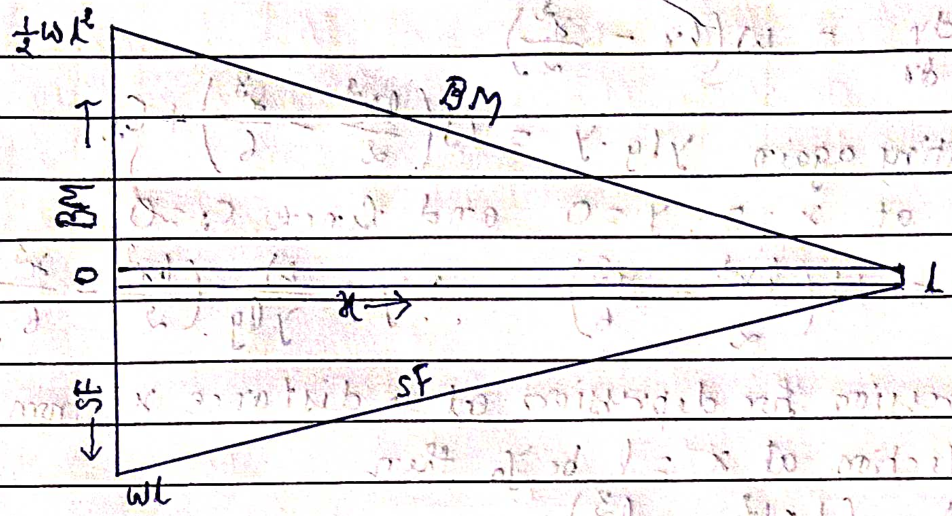
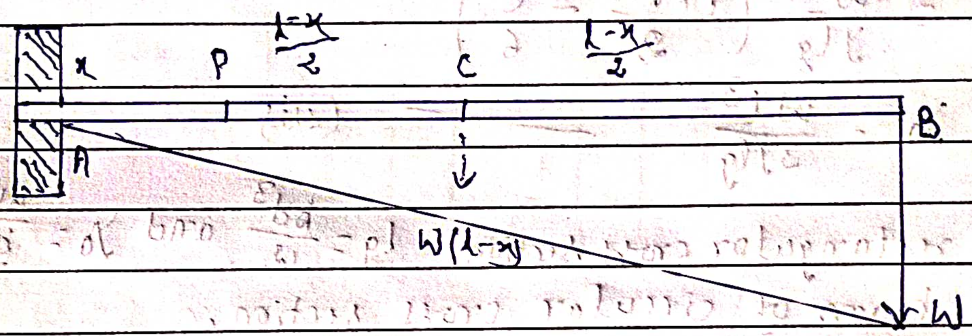
Cantilever :—

Cantilever is a beam fixed horizontally at one end and loaded at the other end.





Case I :— When the wt. of the cantilever does not produce any flexure of the beam. Let the beam AB is fixed at the end A and is loaded with a wt. W at the end B. Let us consider cross-section P at a distance x from the fixed end A. The moment of the external couple at this section due to the load W or the bending moment $= W(l-x) = \frac{Y}{R} I_g$ —(i)

Since the beam is in equilibrium. Here R is the radius of curvature of the natural axis at P and I_g the geometrical moment of inertia of the beam. Since the moment of the load goes on increasing as we go towards A, the radius of curvature goes on decreasing as a consequence

$$\frac{Y I_g}{R} = Y I_g \cdot \frac{d^2 y}{dx^2} = W(l-x) \qquad \text{—— (ii)}$$

and the hence in the case of cantilever, we have case of non uniform bending. Now substituting

$$Y I_g \cdot \frac{1}{R} = Y I_g \cdot \frac{d^2y}{dx^2} = W(l-x)$$

On Integrating, $Y I_g \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right) + C_1$

Now at $x = 0$, $\frac{dy}{dx} = 0$ and hence $C_1 = 0$

$$\therefore Y I_g \frac{dy}{dx} = W\left(lx - \frac{x^2}{2}\right)$$

On Integrating again $Y I_g \cdot y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_2$

Now at $x = 0$, $y = 0$ and hence $C_2 = 0$

$$\therefore Y I_g y = W\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \quad \therefore y = \frac{W}{Y I_g}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right)$$

is the expression for depression at a distance x from fixed end.

If the deflection at $x = l$ be y_0 then

$$y_0 = \frac{W}{Y I_g}\left(\frac{l \times l^2}{2} - \frac{l^3}{6}\right)$$

$$\therefore y_0 = \frac{W l^3}{3 Y I_g} \quad \text{—— (iii)}$$

For a rectangular cross section $I_g = \frac{bd^3}{12}$ and $y_0 = \frac{Wl^3}{\frac{bd^3}{12}} = \frac{4Wd^3}{bd^3 Y}$ —— (iv)

For a beam of circular cross section,

$$I_g = \frac{\pi a^4}{4} \text{ and } y_0 = 3Y \frac{Wl^3}{\frac{\pi a^4}{4}} = \frac{4Wl^3}{3\pi a^4 Y} \quad \text{—— (v)}$$

The young modulus Y can then determined by noting the deflection of the free end of the Cantilever

$$Y = \frac{4Wl^3}{bd^3 y_0} \text{ (For rectangular beam) —— (vi)}$$

$$Y = \frac{4Wl^3}{3\pi a^4 y_0} \text{ (For circular beam) —— (vii)}$$